林同學	回饋建議
書審老師A	能用MIT線上教材自學微積分非常難能可貴, 值得鼓勵! 學習筆記非常出色, 可看出學習歷程 可增加微積分的應用將數學概念和實際應用結合將能更深入體會微積分各單元的深層涵義, 人工智慧/深度學習用到非常多微積分的觀念, 建議可將學到的微積分概念用於探究AI的運作原理
書審老師B	整體歷程表現相當豐富且詳實。是非常優質的學習歷程範本。唯困難點可以從學科的內容及學習規劃兩方面在稍做加強。
書審老師C	勇於挑戰自己，值得嘉許

SELF-LEARN

CALCULUS

111學年度

Massachusetts Institute of Technology

國立宜蘭高中
林同學

edX

CONTENTS

百字簡述

升高中暑假，我開始在 edX 自學 MIT 的 **18.01x 單變量微積分**。在8個月裡我完成了課程、做了**課程筆記**、**報考AP考試**、開始了自動微分的**論文探討**。這些對我的數學能力、自學能力、英文能力、研究能力有了提升。

MOTIVATION

國中時，我因緣際會地翻開了高中的數學課本，當時的我立刻就被叫做「微積分」的數學概念所吸引，在之後的時間裡，我牙牙學語地模仿著課本上的解題方法，只懂得做出課本上的題目而不了解背後的數學原理。

到了升高中的暑假，我決定要在網路上自學大一的微積分，而當我 edX 上發現 MIT 的 18.01x 課程時，我知道我找到了**開啟自學之旅的鑰匙**。

它是個與MIT校內課程「18.01 Single Variable Calculus」相同的 edX XSeries 線上課程。裡面的課堂影片、文章、圖片、互動式題目、測驗等，都促使著我**主動走進微積分的世界**。

PROGRESS

edX XSeries
18.01x Single Variable Calculus

06/23/2022 ~ 08/30/2022
Calculus 1A: Differentiation

09/14/2022 ~ 12/21/2022
Calculus 1B: Integration

02/16/2023 ~ 04/28/2023
Calculus 1C: Coordinate systems & Infinite series

My Study Notes

05/08/2023
AP Calculus

03/2023, 07/2023 ~ Now
ADEV Paper Study

EDX XSERIES
18.01X SINGLE VARIABLE CALCULUS

此課程是與MIT校內課程「18.01 Single Variable Calculus」**相同**的 edX XSeries 線上課程(現已下架)。內容涵蓋單變量微積分的基本概念，如極限、導數和積分，旨在讓學生在學習理論的同時，還能獲得實踐能力，並深入理解微積分的數學符號、物理意義和幾何意義。課程通常通過**校內課堂實拍講座**、**作業**和**測驗**相結合的方式進行學習。

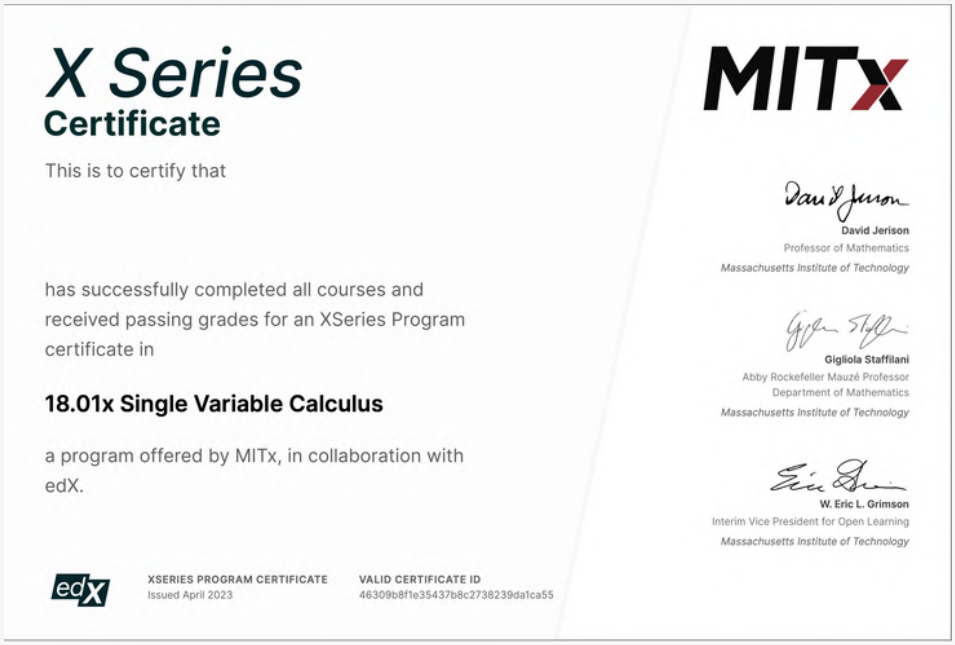
課程內容包含:

Calculus 1A: Differentiation

Calculus 1B: Integration

Calculus 1C: Coordinate systems & Infinite series

與需多線上課程不同，18.01x 是從MIT校內課程 18.01 (單變量微積分)中提取內容，當作 18.01x 的基礎課綱，並結合課堂影片、文章、圖片、互動式題目、測驗等，構建一個高度互動的學習體驗。


X Series
Certificate

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18.01x Single Variable Calculus

a program offered by MITx, in collaboration with edX.

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XSERIES PROGRAM CERTIFICATE
Issued April 2023
VALID CERTIFICATE ID
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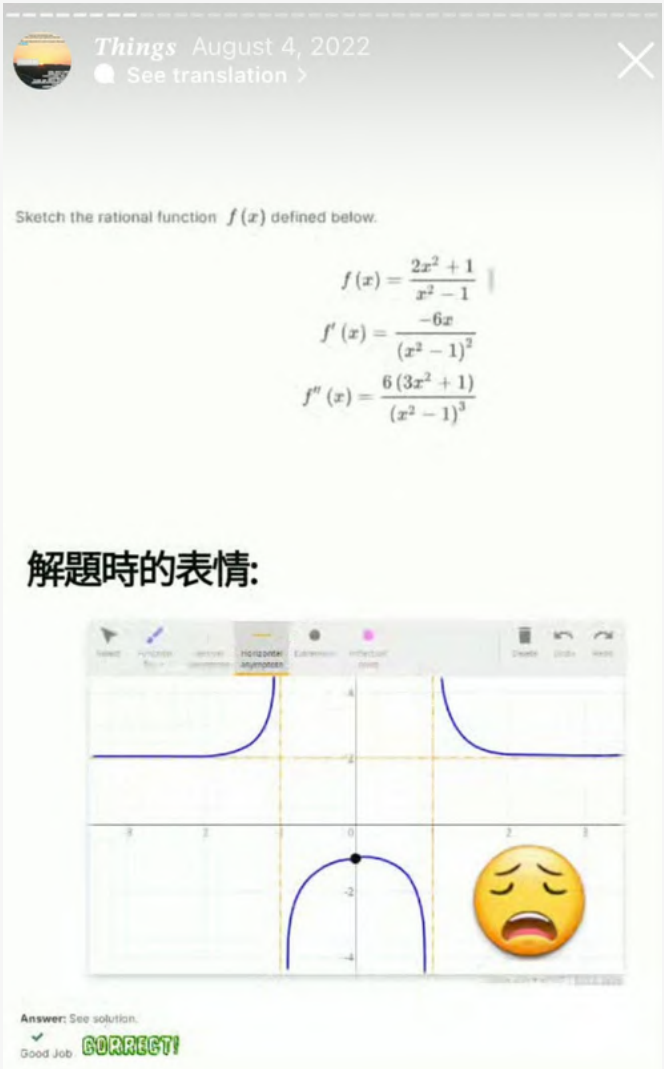
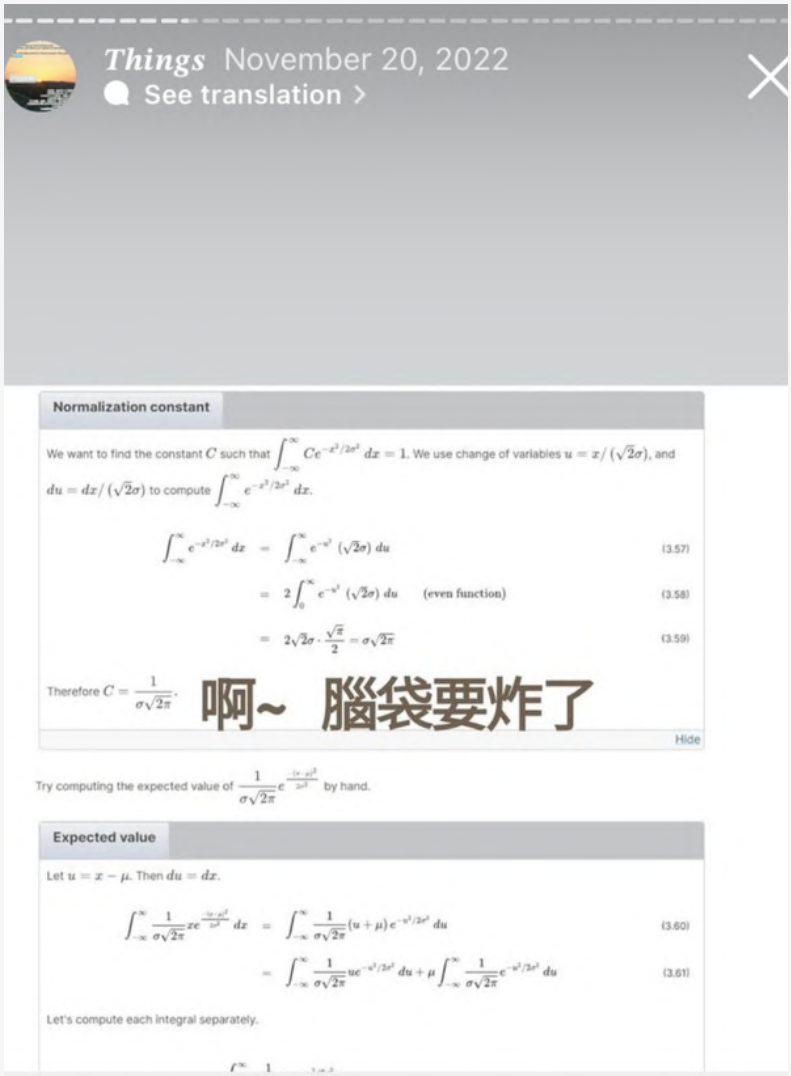
EDX XSERIES
18.01X SINGLE VARIABLE CALCULUS

由於 18.01x 是 **instructor-led** 而非 self-paced，因此我必須要跟著課程所安排的進度學習，在期限前完成所規定的測驗，否則將拿不到該測驗分數。進度的壓力使我寒暑假每周大約會花約**10小時**學習，平日每周則利用假日花**4小時**學習。也因為許多測驗難度很高，有時一道題要花2個多小時才解得出來。



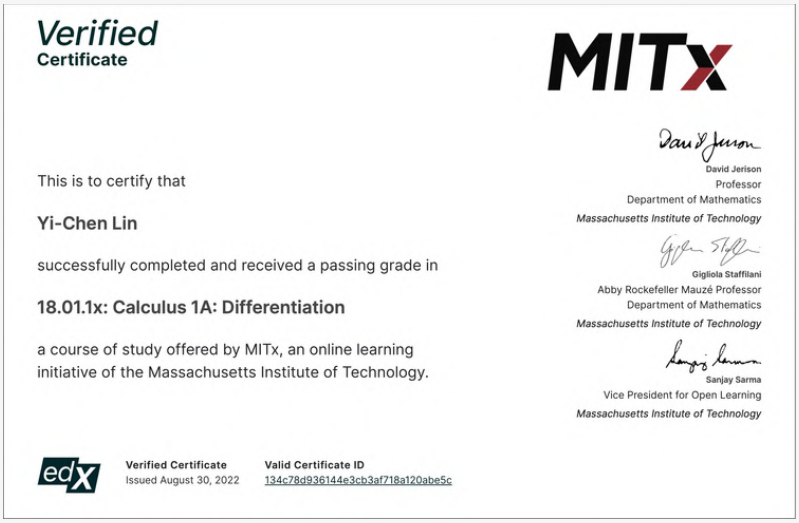
上圖截自 08/04/2022 IG限時動態。



上圖截自 11/20/2022 IG限時動態。

在超過半年的時間裡，我逐漸對於全英文的大學課堂有了一定的熟悉感，透過許多的測驗也使我對於單變量微積分有了一定的掌握。8個月的時間，說長不長，說短不短，但這樣的經歷對我的影響已無法單單用證書呈現。

CALCULUS 1A: DIFFERENTIATION



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Yi-Chen Lin

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18.01.1x: Calculus 1A: Differentiation

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edX Verified Certificate
Issued August 30, 2022

Valid Certificate ID
134c78d936144e3cb3af718a120abe5c

DATE **06/23/2022 ~ 08/30/2022**

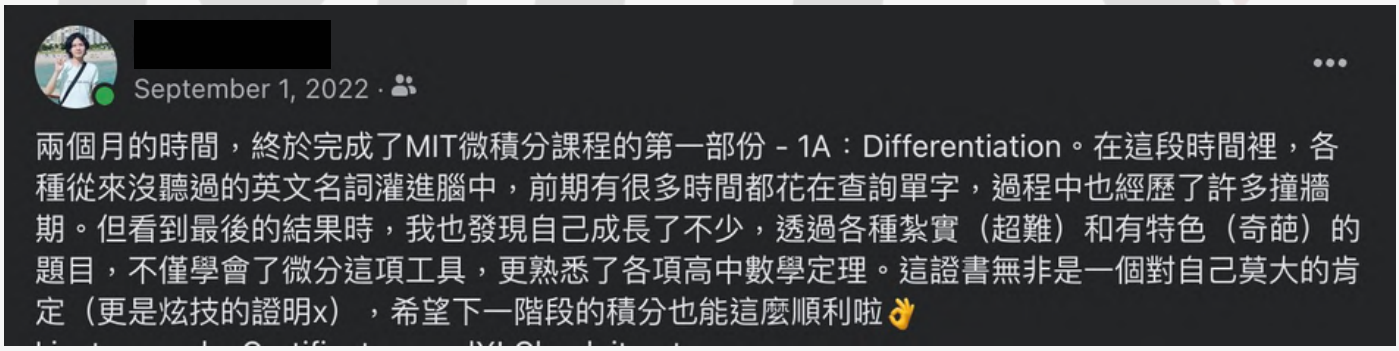
EFFORT **10 hours/week**

DIFFICULTY ★★★☆☆

證書認證: https://courses.edx.org/certificates/134c78d936144e3cb3af718a120abe5c

What I have learned:

- How to evaluate limits **numerically & graphically**
- The **physical meaning**, and **geometric interpretation** of the derivative
- To sketch many functions by hand
- To calculate the derivative of any function
- To apply derivatives to **maximize & minimize functions** and find **related rates**
- To make **linear & quadratic approximations** of functions



[redacted]
September 1, 2022 ·

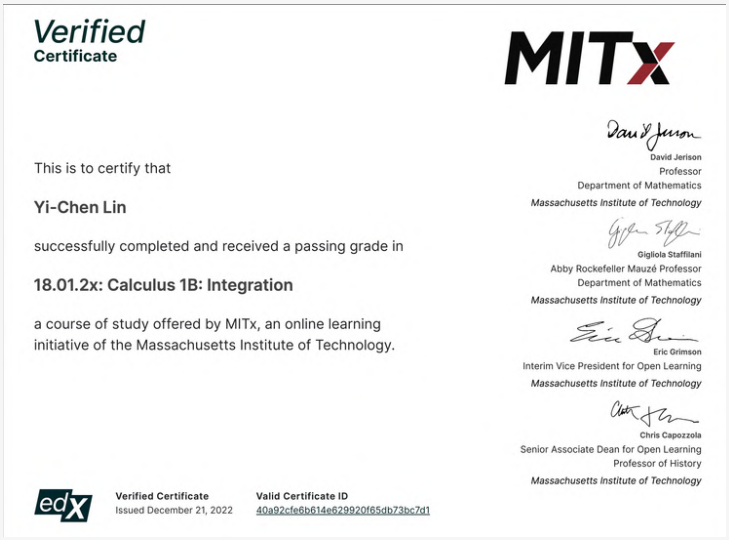
兩個月的時間，終於完成了MIT微積分課程的第一部份 - 1A：Differentiation。在這段時間裡，各種從來沒聽過的英文名詞灌進腦中，前期有很多時間都花在查詢單字，過程中也經歷了許多撞牆期。但看到最後的結果時，我也發現自己成長了不少，透過各種紮實（超難）和有特色（奇葩）的題目，不僅學會了微分這項工具，更熟悉了各項高中數學定理。這證書無非是一個對自己莫大的肯定（更是炫技的證明x），希望下一階段的積分也能這麼順利啦👌

上圖截自我於 09/01/2022 發布的 FB 貼文。

Calculus 1A 是 18.01x 中的第一個課程，也是我的第一個全英文線上課程。課程中有許多測驗是採用引導的方式鼓勵學員自己推導出公式及解法，因此已經習慣套用現成的公式的我一時之間感到不太適應。

不過也因為這些挑戰加上進度壓力，使我更加投入學習。這個大學課程初體驗無疑對我在日後的科展研究與未來的學習之路產生了深遠的影響。其中關於連鎖律的知識更是對我在閱讀一篇關於**自動微分應用於概率編程(Probabilistic Programming)** 的論文有了一定的幫助。

CALCULUS 1B: INTEGRATION



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18.01.2x: Calculus 1B: Integration

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Verified Certificate
Issued December 21, 2022

Valid Certificate ID
40a92cfe6b614e629920f65db73bc7d1

DATE **09/14/2022 ~ 12/21/2022**

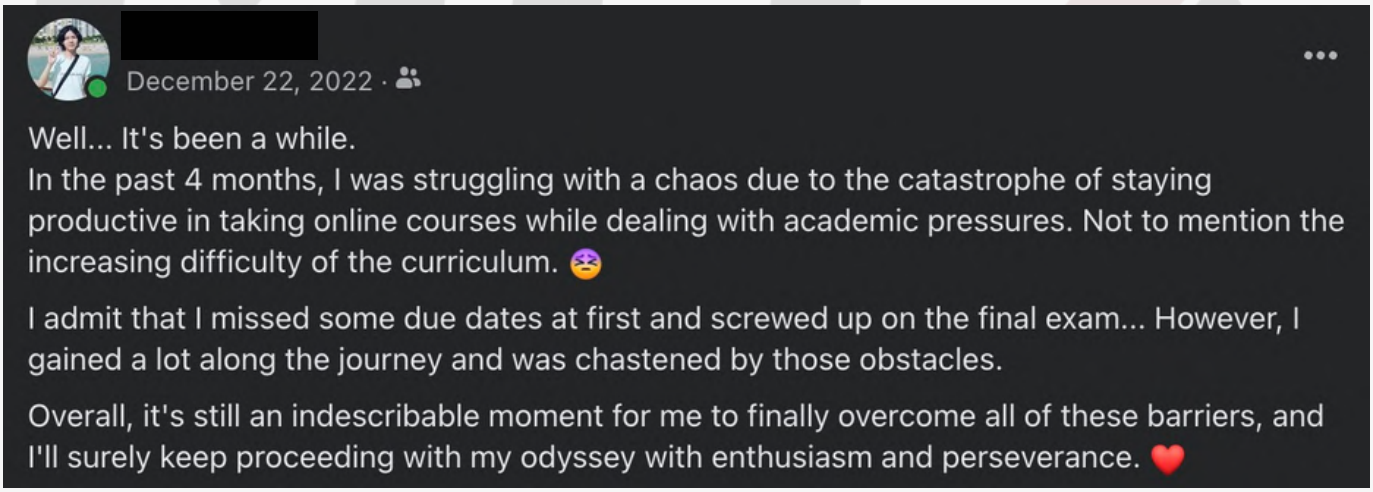
EFFORT **6 hours/week**

DIFFICULTY ★★★★½

證書認證: https://courses.edx.org/certificates/40a92cfe6b614e629920f65db73bc7d1

What I have learned:

- Some **differential equation models** for physical phenomena and solutions
- The **geometric interpretation** of the integral as an area under a graph, and the **physical meaning** as an accumulation
- Several methods of **numerically** and **symbolically** integrating functions
- The connection of the integral to the derivative
- To apply integrals to solve **real-world problems**



[redacted]
December 22, 2022 ·

Well... It's been a while.
In the past 4 months, I was struggling with a chaos due to the catastrophe of staying productive in taking online courses while dealing with academic pressures. Not to mention the increasing difficulty of the curriculum. 😫

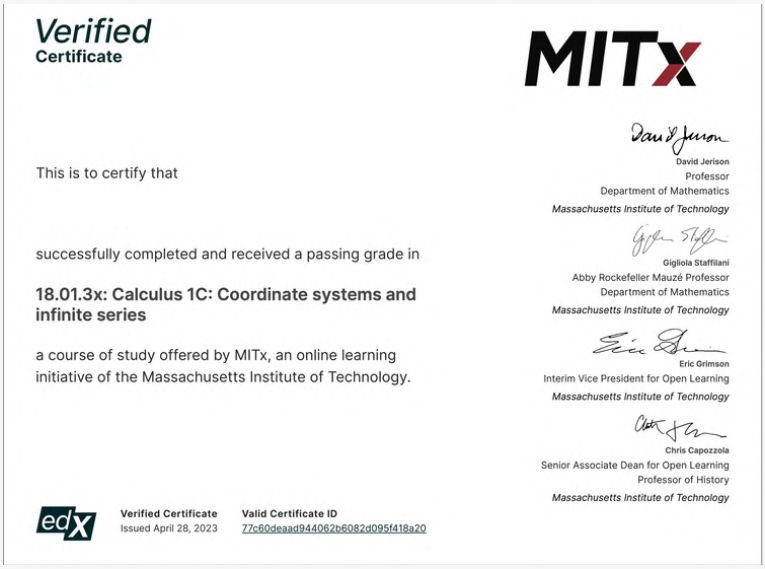
I admit that I missed some due dates at first and screwed up on the final exam... However, I gained a lot along the journey and was chastened by those obstacles.

Overall, it's still an indescribable moment for me to finally overcome all of these barriers, and I'll surely keep proceeding with my odyssey with enthusiasm and perseverance. ❤️

上圖截自我於 12/22/2022 發布的 FB貼文。

這門課程的難度著實讓我感到有些吃力。特別是各種積分技巧和概念，而這也直接影響了我的課程分數。不過，每每解開一個難題，心中的成就感總是無法言喻。這次的經歷也讓我更了解自己的學習方式，也知道該如何更有效地學習與改進。

CALCULUS 1C: COORDINATE SYSTEMS & INFINITE SERIES



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18.01.3x: Calculus 1C: Coordinate systems and infinite series

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edX Verified Certificate
Issued April 28, 2023

Valid Certificate ID
77c60deaad944062b6082d095f418a20

DATE 02/16/2023 ~ 04/28/2023

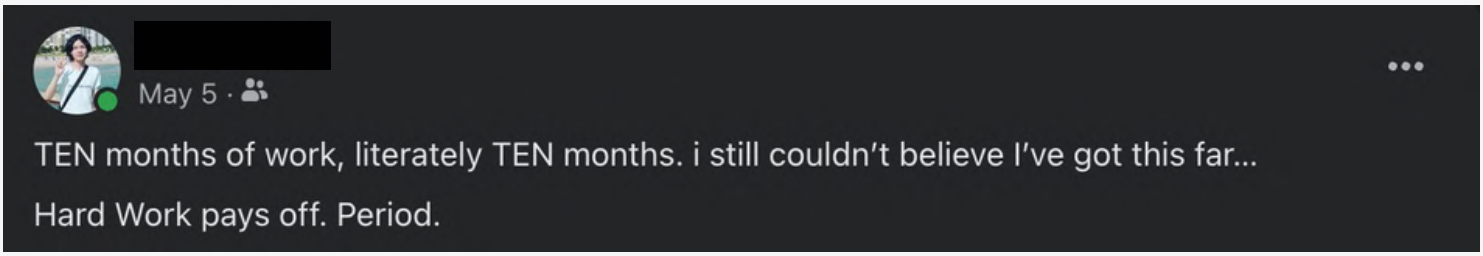
EFFORT 6 hours/week

DIFFICULTY ★★★★☆

證書認證: https://courses.edx.org/certificates/77c60deaad944062b6082d095f418a20

What I have learned:

- To compute **arc length**
- To do calculus in **polar coordinates**
- Methods for **parameterizing curves**, and using the parameterizations to solve problems
- How to approximate functions with **Taylor polynomials & series**
- To determine convergence properties of **infinite series**



上圖截自我於 05/05/2023 發布的 FB 貼文。

Calculus 1C 是 18.01x 中的最後一個課程，也同樣是一門具有挑戰性的課程。不管是極坐標系還是泰勒展開，都是我十分陌生的內容，且需要將先前的1A與1B的課程融會貫通。不過在學習的過程中，我也能夠明顯地感覺到自己在經過前兩個課程的洗禮過後的自我成長，不論是**英文能力**、**數學思維**、**計算能力**、**自學能力**都有明顯的進步。

MY STUDY NOTES

Derivative

Definition

$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ $\quad f''(x)=\frac{d}{dx}\frac{dy}{dx}=\frac{d^2y}{dx^2}=\frac{\frac{dy}{dx}}{\frac{dx}{dt}}$

List

$\sin'(x)=\cos(x)$

$\cos'(x)=-\sin(x)$

$\tan'(x)=\sec^2(x)$

$\sec'(x)=\tan(x)\sec(x)$

$\arcsin'(x)=\frac{1}{\sin'(\arcsin(x))}=\frac{1}{\cos(\arcsin(x))}=\frac{1}{\sqrt{1-x^2}}$

$\frac{de^x}{dx}=e^x$ $\quad \frac{d}{dx}\ln x=\frac{1}{x}$ $\quad \frac{d}{dx}a^x=a^x\ln a$

Product Rule

$\frac{d}{dx}f(x)g(x)=f'(x)g(x)+f(x)g'(x)$

Chain Rule

$f'(g(x))=f'(g(x))\cdot g'(x)$

Quotient Rule

$\frac{d}{dx}\frac{f(x)}{g(x)}=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$

Inverse Function

$f^{-1}(x)=g(x)$ $\quad g'(x)=\frac{1}{f'(g(x))}$

Linear Approximation

$f(a)\approx f(x)+f'(x)(a-x)$

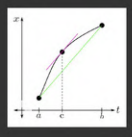
Quadratic Approximation

$f(a)\approx f(x)+f'(x)(a-x)+\frac{f''(x)}{2}(a-x)^2$

Mean Value Theorem (MVT)

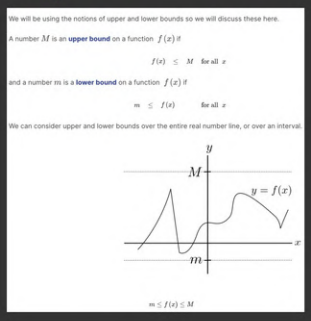
Somewhere in $[a,b]$, there is a tangent line that is parallel to the secant line

$\frac{f(b)-f(a)}{b-a}=f'(c)$



If $x(t)$ is continuous in $[a,b]$ and differentiable on (a,b)

Upper and lower bounds



Separation of variables

$\frac{dy}{dx}=f(x)g(y)$ $\quad \frac{dy}{g(y)}=f(x)dx$

$\int\frac{dy}{g(y)}=\int f(x)dx$

E.g. $\frac{dy}{dx}=xy^2$, $y_0=?$

$\frac{dy}{y^2}=x\,dx\Rightarrow\int\frac{dy}{y^2}=\int x\,dx\Rightarrow -\frac{1}{y}=\frac{1}{2}x^2+C$

$\Rightarrow y=-\frac{1}{\frac{1}{2}x^2+C}$ ※

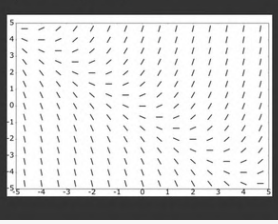
$\frac{dy}{dx}=(2x+5)^4$, $y=?$

$\int dy=\int(2x+5)^4dx\Rightarrow y=\frac{1}{10}(2x+5)^5+C$ ※

Slope Fields

At each point (x,y), draw a short segment whose slope is the value of y' at the point (x,y)

E.g. The slope field of $\frac{dy}{dx}=x+y$:



Integral

Antiderivative / Indefinite integral

$\int f(x)dx=F(x)+\underbrace{C}_{\text{any constant}}$

$\int x^p dx=\frac{x^{p+1}}{p+1}+C$ ※ $\int\frac{dx}{x}=\ln(|x|)+C$

Integration Rule

$\int kf(x)dx=k\int f(x)dx$

$\int(f(x)+g(x))dx=\int f(x)dx+\int g(x)dx$

E.g. Compute the antiderivatives:

$\int\frac{3}{2x}dx$?

A: $3\ln(|x|)+C$

$\int 4x(5x^2-1)^{\frac{1}{3}}dx$: let $u_1=5x^2-1$ $du_1=10x\,dx$

$=\int\frac{2}{5}du_1\cdot(u_1)^{\frac{1}{3}}$

$=\frac{2}{5}\frac{3}{4}(u_1)^{\frac{4}{3}}+C=\frac{3}{10}(5x^2-1)^{\frac{4}{3}}+C$

A: $\frac{3}{10}(5x^2-1)^{\frac{4}{3}}+C$

$\int e^{2x}\cos(1-e^{2x})dx$: let $u=1-e^{2x}$ $du=-2e^{2x}dx$

$=\int-\frac{1}{2}du\cdot\cos(u)$

$=-\frac{1}{2}\sin(u)+C=-\frac{1}{2}\sin(1-e^{2x})+C$

A: $-\frac{1}{2}\sin(1-e^{2x})+C$

$\int\tan x\,dx$: let $u_2=\cos x$ $du_2=-\sin x\,dx$

$=\int\frac{\sin x}{\cos x}dx$

$=\int\frac{-du_2}{u_2}$

$=-\ln(|u_2|)+C=-\ln(|\cos x|)+C$

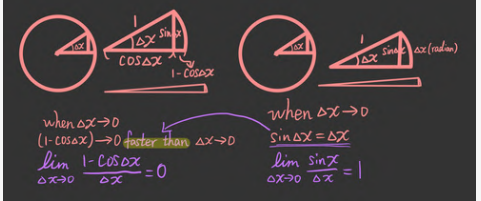
A: $-\ln(|\cos x|)+C$

The derivative of $\sin x$

$\frac{d}{dx}\sin x=\lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$

$=\lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x+\cos x\sin\Delta x-\sin x}{\Delta x}$

$=-\sin x\left(\lim_{\Delta x\to 0}\frac{1-\cos\Delta x}{\Delta x}\right)+\cos x\left(\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}\right)$



when $\Delta x\to 0$, $(1-\cos\Delta x)\to 0$ faster than $\Delta x\to 0$

$\lim_{\Delta x\to 0}\frac{1-\cos\Delta x}{\Delta x}=0$

when $\Delta x\to 0$, $\sin\Delta x=\Delta x$

$\lim_{\Delta x\to 0}\frac{\sin x}{\Delta x}=1$

$=\cos x$ ※

Differentials

$y=F(x)$, $\frac{dy}{dx}=F'(x)\to dy=F'(x)dx$

Linear approximation: $f(x+dx)\approx y+dy$

E.g.

Approximate $\sqrt{21}$ by using a linear approximation to $f(x)=\sqrt{10x-x^2}$ based at $x_0=2$. (Use differentials)

$f(2)=4$ $\quad f(3)=\sqrt{21}$ $\quad f'(x)=(10-2x)\cdot\frac{1}{2}(10x-x^2)^{-\frac{1}{2}}$

$f'(2)=6\times\frac{1}{8}=\frac{3}{4}$ $\quad dx=1$

$f(2+1)\approx 16+\frac{3}{4}\cdot 1$

A: $16\frac{3}{4}$

Euler's Method

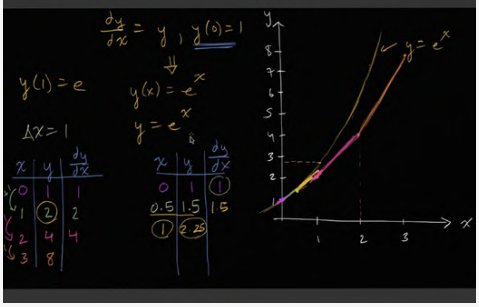
1. Choose a step size Δx (the smaller the more accurate)
2. Let (x_0,y_0) be the initial condition
3. $x_1=x_0+\Delta x$

$y_1=y_0+\Delta y=y_0+y'(x_1)\cdot\Delta x$ $\because\frac{dy}{dx}=\frac{\Delta y}{\Delta x}\therefore\Delta y\approx\frac{dy}{dx}$

4. Iterate the process... (linear approximation)

$x_{k+1}=x_k+\Delta x$

$y_{k+1}=y_k+y'(x_{k+1})\cdot\Delta x$ (linear approximation)



MY STUDY NOTES

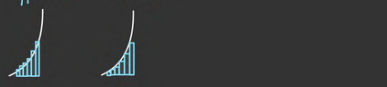
Summation Notation (Sigma)

$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_{n-1} + a_n$ "the sum of a_i from $i=1$ to $i=n$."

Riemann Sum

$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$

upper sum lower sum



E.g. Write $\int_1^4 (x^2-2)dx$ as the limit of a Riemann Sum

$\Delta x = \frac{3}{n}$ $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(1+\frac{3k}{n}\right)^2-2\right)\frac{3}{n}$ #

E.g. Write $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{5k}{n}\right)^2 \cdot \frac{5}{n}$ as a definite integral

$\Delta x = \frac{5}{n} = \frac{b-a}{n}$, $b=7$ $\int_2^7 x^2 dx$ #

Function Of Error

$\text{erf}(x) = \frac{\pi}{\sqrt{2}} \int_0^x e^{-t^2}$

Trapezoids

$T(n) = \frac{b_0+b_1}{2}\cdot h_1 + \frac{b_1+b_2}{2}\cdot h_2 + \cdots + \frac{b_{n-1}+b_n}{2}\cdot h_n$

Simpson's Method

$\frac{\Delta x}{6}\left(x_0+2x_1+4y_2+2x_3+\cdots+4x_{n-1}+2x_{n-1}+x_n\right)$

Integral with odd positive exponent

recall: $\cos^2 = 1-\sin^2(x)$

$\sin^2 = 1-\cos^2(x)$

Integral with even positive exponent

recall: $\sin^2(x) = \frac{1-\cos(2x)}{2}$

$\cos^2(x) = \frac{1+\cos(2x)}{2}$

E.g. $\int \sin^3(x)\cos^2(x)dx = ?$

$= \int (1-\cos^2(x))\sin(x)\cos^2(x)dx$

$= \int (\cos^2(x)-\cos^4(x))\sin(x)dx$

let $u=\cos(x)$, $du = -\sin(x)dx$

$= \int -(u^2-u^4)du = \int (u^4-u^2)du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3} + C$ ※

E.g. $\int \sin^2(x)dx = ?$

$= \int \frac{1-\cos(2x)}{2}dx$

$= \frac{x}{2} - \frac{\sin(2x)}{2\cdot 2} + C$

Integration by parts

$\int uv'dx = uv - \int u'v\,dx$

※ COMPLEX Function Usually Use v'

Reduction Formula

$I_n = \int x^p (\ln x)^n dx$

$= \frac{(\ln x)^n x^{p+1}}{p+1} - \frac{n}{p+1} I_{n-1} + C$

$G_n(x) = \int x^n e^x dx$

$= x^n e^x - nG_{n-1}(x)$

Arc Length Integral

$ds = \sqrt{dx^2+dy^2} = \sqrt{(f'(x))^2+1}\,dx$

arclength $(0\le x\le a)$

$= \int_0^a ds$

Surface Area

rotated by x-axis: $\int_a^b 2\pi y\,ds$

rotated by y-axis: $\int_a^b 2\pi x\,ds$

E.g. $\int \frac{x^5}{(x^3+1)^5}dx = ?$

let $u = x^5$ $v' = (x^3+1)^{-5}$,

$u' = 5x^4$ $v = \frac{1}{-12}(x^3+1)^{-4}$

$\int uv'dx = uv - \int u'v\,dx$

$= -\frac{x^5}{12}(x^3+1)^{-4} - \int -\frac{5}{12}x^4(x^3+1)^{-4}dx$

$\vdots$

$= -\frac{x^5}{12}(x^3+1)^{-4} - \frac{1}{36}(x^3+1)^{-3}$ ※

E.g. Find area between $x=y^2$ and $y=x-2$

Method 1

Step 1. draw a picture

Step 2. find the intersection points $x = x^2-4x+4$ $x^2-5x+4=0$

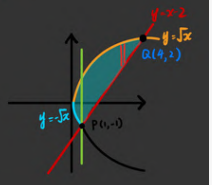
$x=4$ $y=2$; $x=1$ $y=-1$

Step 3. identify each curves

Step 4. calculate area

Area $= \int_0^1 (\sqrt{x}-(-\sqrt{x}))dx + \int_1^4 (\sqrt{x}-(x-2))dx$

left right

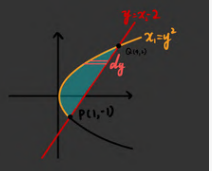


Method 2 (better)

use horizontal slices

...

Area $= \int_{-1}^{2} (y+2-y^2)dy$



Trig Substitutions

if: $\sqrt{x^2-a^2} \to a\tan\theta$

$\sqrt{a^2-x^2} \to a\sin\theta$

$\sqrt{x^2+a^2} \to a\sec\theta$

E.g. $\int \frac{dx}{\sqrt{(2x^2+8)^3}} = ?$

$= \frac{1}{2\sqrt{2}} \int \frac{dx}{\sqrt{(x^2+4)^3}}$

$= \frac{1}{2\sqrt{2}} \int \frac{3\sec^2\theta}{3^3\sec^3\theta}d\theta$ $\because x = 3\tan\theta$, $dx = 3\sec^2\theta\,d\theta$

$= \frac{1}{18\sqrt{2}} \int \frac{1}{\sec\theta}d\theta$

$= \frac{1}{18\sqrt{2}} \int \cos\theta\,d\theta = \frac{1}{18\sqrt{2}\sin\theta} = \frac{x}{18\sqrt{2}\sqrt{x^2+9}} + C$

E.g. $\int \frac{dx}{\sqrt{4x^2-1}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{x^2-\frac{1}{4}}}$

$= \frac{1}{2}\int \frac{\sec\theta\tan\theta}{\tan\theta}d\theta$ $\because x = \frac{1}{2}\sec\theta$, $dx = \frac{1}{2}\sec\theta\tan\theta\,d\theta$

$= \frac{1}{2}\int \sec\theta\,d\theta$

$= \frac{1}{2}\int \sec\theta\cdot\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}d\theta$

$= \frac{1}{2}\int \frac{\sec^2\theta+\sec\theta\tan\theta}{\sec\theta+\tan\theta}d\theta$ $u = \sec\theta+\tan\theta$, $du = \sec\theta\tan\theta+\sec^2\theta$

$= \frac{1}{2}\int \frac{1}{u}du$

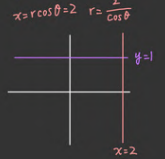
$= \frac{1}{2}\ln|u| = \frac{1}{2}\ln(|\sec\theta+\tan\theta|)$ ※

Polar Coordinate Arc Length Integral

$\int \sqrt{(r(\theta))^2 + r'(\theta)^2}\,d\theta$

$y = r\sin\theta = 1$ $r = \frac{1}{\sin\theta}$

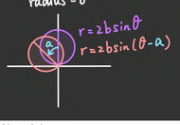
$x = r\cos\theta = 2$ $r = \frac{2}{\cos\theta}$



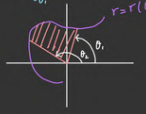
$(x-b)^2+y^2 = b^2$, $r = 2b\cos\theta$

$x^2+(y-b)^2 = b^2$, $r = 2b\sin\theta$

radius $= b$



$A = \int_{\theta_0}^{\theta_2} dA = \frac{1}{2}\int_{\theta_0}^{\theta_2} r(\theta)^2 d\theta$



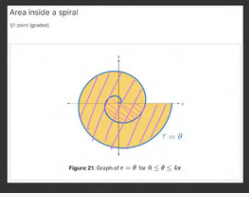
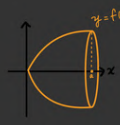


Figure 21: Graph of $r=\theta$ for $0\le\theta\le 4\pi$

$A = \frac{1}{2}\int_0^{4\pi} \theta^2 d\theta - \frac{1}{2}\int_0^{2\pi} \theta^2 d\theta$

$= \frac{1}{2}\int_{2\pi}^{4\pi} \theta^2 d\theta$

Volume



$V = \int_0^a \pi y^2(x)\,dx$

$V = \int_0^4 4\pi xy\,dy$

Average value of f(x)

$\text{ave}(f(x)) = \frac{1}{b-a}\int_a^b f(x)dx$

Weighted average

$\frac{\int_a^b f(x)w(x)dx}{\int_a^b w(x)dx}$

Center of mass (1D)

$\bar{x} = \frac{\int_0^l x\cdot\lambda(x)dx}{\int_0^l \lambda(x)dx}$

Center of mass (2D)

$\bar{x} = \frac{\int_a^b x\cdot dA}{\int_a^b dA}$ $dA = ydx$

$\bar{y} = \frac{\int_a^b y\cdot dA}{\int_a^b dA}$ $dA = x\,dy$

Partial Fraction

E.g. $\int \frac{4x-1}{x^2+x-2}dx = ?$

$= \int \frac{4x-1}{(x-1)(x+2)}dx$

$= \int \frac{A}{x-1} + \frac{B}{x+2}dx$ for A: $\times(x-1)$; for B: $\times(x+2)$

$\int A + \frac{B(x-1)}{x+2}dx = \int \frac{4x-1}{x+2}$

if $x=1$, $A=1$

$\int \frac{A(x+2)}{x-1} + B = \int \frac{4x-1}{x-1}$

if $x=-2$, $B=3$

$= \int \frac{1}{x-1} + \frac{3}{x+2}dx$

$= \ln(|x-1|) + 3\ln(|x+2|) + C$ ※

E.g. $\int \frac{2x-9}{(x^2+9)(x+2)}dx$ if $x=-2$: $\times(x+2)$: $-1 = C$

$\frac{2x-9}{(x^2+9)(x+2)} = \frac{Ax+B}{(x^2+9)} + \frac{C}{(x+2)}$

$\times(x^2+9)(x+2)$: $2x-9 = (Ax+B)(x+2) - x^2 - 9$ $A-1=0$, $A=1$; $2B-9=-9$, $B=0$

$= \int \left(\frac{x}{x^2+9} - \frac{1}{x+2}\right)dx = \int \frac{x}{x^2+9}dx - \int \frac{1}{x+2}dx$

let $u = x^2+9$, $du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{u}du - \ln(|x+2|)$

$= \frac{\ln(|x^2+9|)}{2} - \ln(|x+2|) + C$

Infinity

for $a>0$, $\int_a^\infty \frac{dx}{x^p}$ { diverges if $p\le 1$; converges to $\frac{a^{-p+1}}{p-1}$ if $p>1$

Absolute / Conditional convergence

Absolute: if $\sum_{n=1}^{\infty} |a_n|$ converges

Conditional: if it converges but is NOT Absolute convergent

Ratio test

$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$L<1 \Rightarrow$ abs converges

$L>1 \Rightarrow$ diverges

$L=1 \Rightarrow$ no conclusion

Root test

$L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

Alternating series

$\pm\sum_{n=1}^{\infty} (-1)^n a_n$

{ $\lim_{n\to\infty} a_n = 0$; a_n Decreases as n increases $\to$ a_n Converges

Taylor's formula

$\frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

AP CALCULUS

在學習微積分的同時，我也在思考:

「我該如何展現我的學習成果呢?」

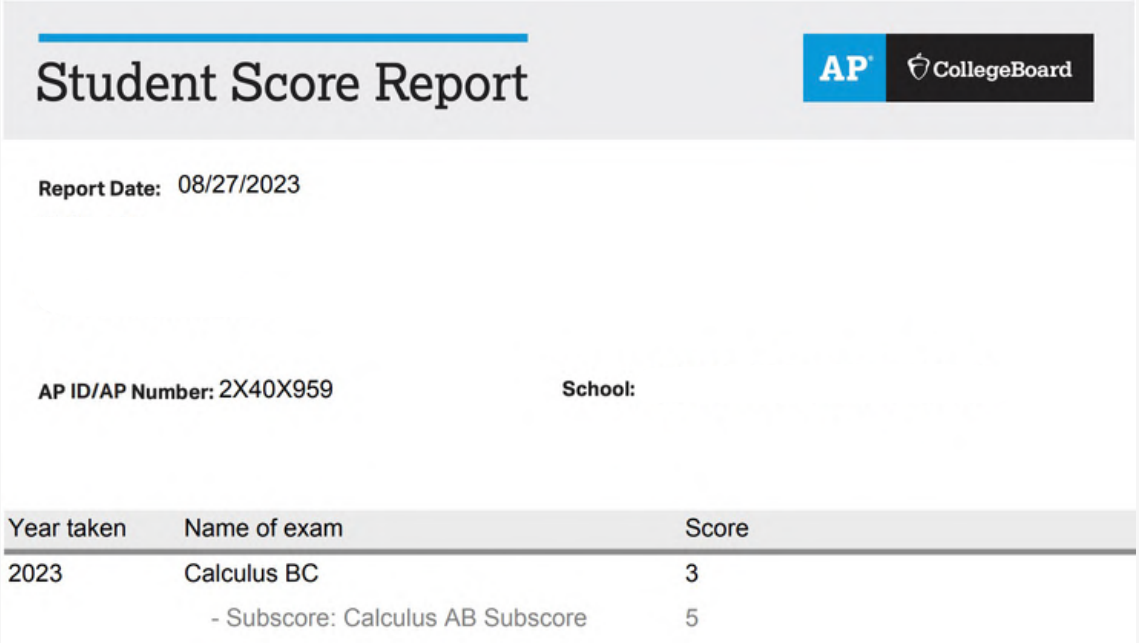
當時的我便上網查詢高中生的微積分證書或考試，剛好找到了由美國大學理事會提供的 AP Calculus 考試。雖然AP考試並沒有對體制外的高中生開放報名，但在主動聯繫了幾間國際學校後還是有成功報名。

美國大學理事會稱:「AP微積分課程需要高中生花費一年來完成，以對應大學所教授的相應科目。」

AP Calculus AB及BC是兩門大學先修課程中的微積分科目。內容涵蓋了臺灣大一上學期及部分下學期的微積分課程。

AP Calculus BC包含以下主題(紅色部分為AB缺少的內容):

- Limits and Continuity
- Differentiation: Definition and Fundamental Properties
- Differentiation: Composite, Implicit, and Inverse Functions
- Contextual Applications of Differentiation
- Analytical Applications of Differentiation
- Integration and Accumulation of Change
- Differential Equations
- Applications of Integration
- Parametric Equations, Polar Coordinates, and Vector-Valued Functions
- Infinite Sequences and Series



Student Score Report

AP CollegeBoard

Report Date: 08/27/2023

AP ID/AP Number: 2X40X959

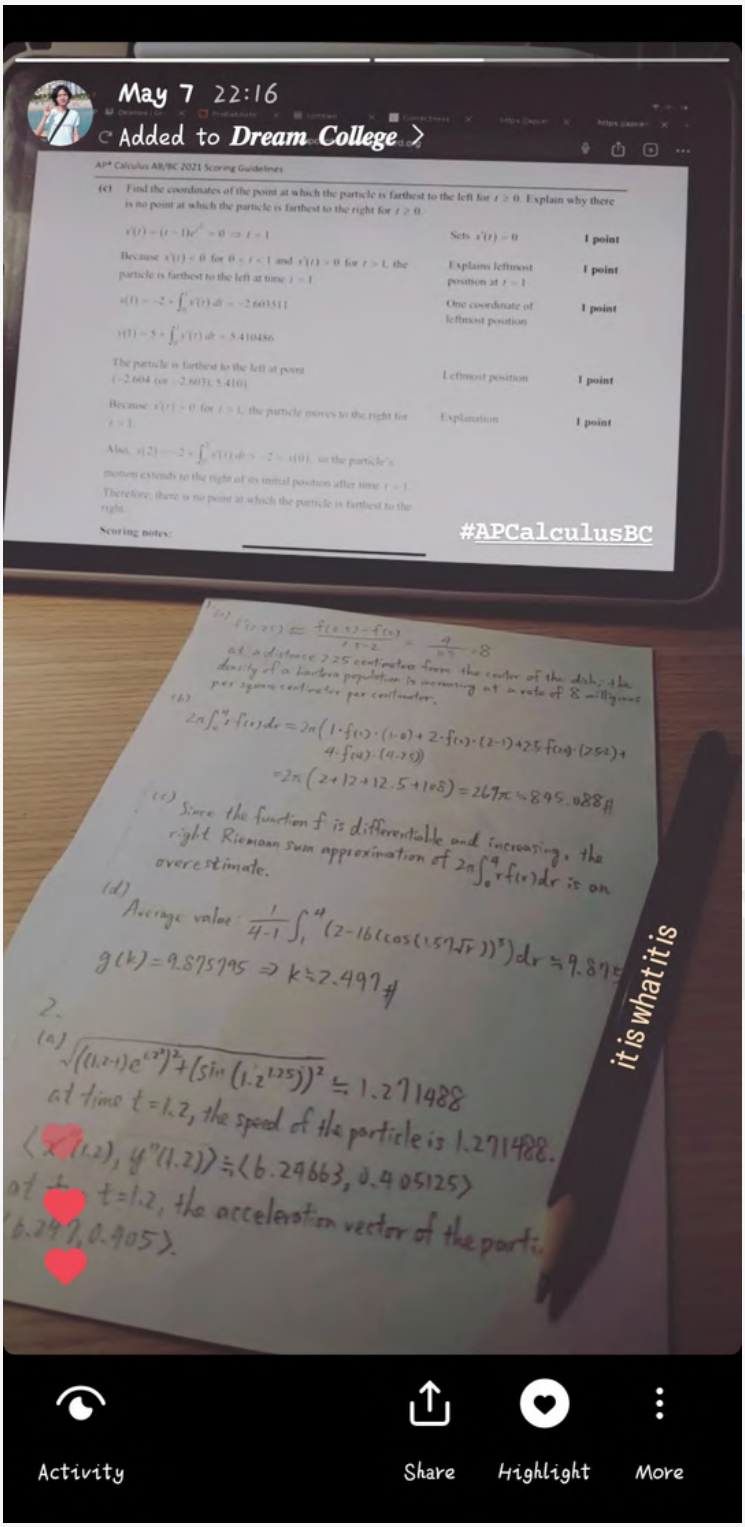
School:

Year taken	Name of exam	Score
2023	Calculus BC	3
	- Subscore: Calculus AB Subscore	5

上圖截自我的AP Calculus BC成績單(BC為3分, AB分項分數則為5分)

AP CALCULUS

為了準備AP考試，我特地去到臺北的書店挑選了一本參考書，全英文+微積分+3.5cm厚的書，在班上拿出來每每成為群眾焦點。考前的我也花了些時間準備非選擇題，並拿出先前製作的筆記複習。



上圖截自 05/07/2023 IG限時動態。

不過即使是這樣，我最終的AP Calculus BC成績為3分(滿分5分)，距離自己所期望的分數還是有些落差。

令人開心的是AP Calculus AB的分項分數有達到滿分5分。間接證明了我對於課程依然具有一定的掌握度。

經過此次考試的洗禮，我了解到了自己對於微積分的部分主題依然生疏(無窮級數與數列、極座標系、向量值函數)，而我對於非選擇題也較無自信。

在之後的時間，我會著重加強我較缺乏掌握度的主題，並持需精進我對於微積分的知識，並期盼能將這個技能運用於考試以外的項目。

ADEV PAPER STUDY

今年三月的某日放學，我的手機彈出了一條通知，是關於麻省理工學院新聞的一篇文章，名為 "**Automating the math for decision-making under uncertainty**"。

強烈的好奇心驅使我閱讀了這篇文章，並藉此找到了一篇論文: "**ADEV - Sound Automatic Differentiation of Expected Values of Probabilistic Programs**"，因而激發了我對於**自動微分(AD)** 及 **概率編程(PP)** 的興趣，也萌生了想要做一份相關的科展作品的念頭。

為了想要讀懂ADEV的論文並進行科展研究，我首先複習了先前在 **18.01x** 所學的**單變量微分**，並自己不斷地搜尋AD與PP的相關資料、觀看**作者介紹ADEV的影片**，甚至**主動聯繫**論文的四位作者，詢問有關論文的問題及科展的方向(其中一人有回覆)。再尋找資料的過程中卻發現這兩個都是十分冷門的領域，概率編程的相關論文也是近年才逐漸受到關注。

Hi Emery,

Nice to meet you and your friends Kevin and Jurgen, and thanks for getting in touch!
I'm glad you got inspired by our research. I'm not sure I liked the Mark TechPost article which seems like a worse version of the original popular article that came out (https://news.mit.edu/2023/automating-math-decision-making-under-uncertainty-0206).

I'm happy to see you are so motivated by the kind of science we're doing, and I highly encourage you to continue on this path. Being proactive and curious like you were is in my opinion one of the best ways to make fast progress, no matter the topic.

I am sorry I won't be able to mentor you directly. I just started a new job at MIT and my cup is pretty full. In addition, I think the kind of research we are doing might be a bit too hard for you and your friends with your **current** background. Again, given your motivation, I highly advise you to push your interest further, e.g. by going to a university where you can learn way more about all the topics you mentioned. To understand properly what we did, you sort of need a degree both in maths and CS, but you can always catch up with the one which is not your major, though maths is usually a bit harder. With your CV, you can hopefully get to a good university with motivating people around you, maybe you can also consider MIT and eventually work with us 🙂.

About your ideas, I think they both make sense. 1 is interesting, but to be honest I'm not sure how I would even start doing that seriously, so you might be underestimating the difficulty, and people already do quite a bit of **data augmentation.** Reducing the size of training sets for ML is a big goal of Probabilistic programming, but again it is much harder than you might think as it requires a deep rethinking of the complex architecture of neural-based models such as GPT or Midjourney, which are already highly optimized. It's definitely doable and will eventually be done in the coming years though.
Your second idea is already being done though. https://github.com/probcomp/genjax is an ongoing project, and it targets Python (Jax is an efficient Python library for writing and training machine learning models, it is the current state of the art, and it is based on functional programming behind the scene). GenJax has a prototype implementation of ADEV (https://github.com/probcomp/genjax/blob/main/src/genjax/_src/core/transforms/adev.py).

Re your questions:

1. ADEV is a research project, and it will take some time before it's as user-friendly as other libraries you might've used. Maybe through GenJax is currently the best way. We are planning on making progress this summer for better accessibility to students.
2. Monte Carlo (MC) algorithms are a class of algorithms for sampling data points from a given distribution. It takes some course to understand the detail, but roughly you can reduce a lot of problems about uncertainties and probabilities to accurately approximate sophisticated integrals, and Markov Chain Monte Carlo (MCMC) algorithms are a subclass of MC that have a particular strategy for doing so (they rely on Markov Chains). Variational Inference is a different strategy where instead of approximating a complex distribution, you sample from a simpler/nicer distribution that you use as a proxy for the more complex one. Variational Inference is then a (class of) algorithm(s) that tell you how to obtain a simpler distribution that is as close as possible to the original complex one. It does so by minimizing a "distance" between the simple distribution and the complex one.
3. It depends. Again, it's a tricky question, they all have pros and cons, and there is no generally great answer. I think Gen(https://www.gen.dev/), the one we are developing at MIT, is one of the most promising ones, and it is being actively developed. So perhaps GenJax is what you are looking for, but we still need to polish it more with better tutorials etc. In the meantime, Pyro(https://pyro.ai/) is nice and supported by the industry, and WebPPL(http://webppl.org/) is nice for learning about PPLs.

上圖截自ADEV論文作者的郵件回覆。

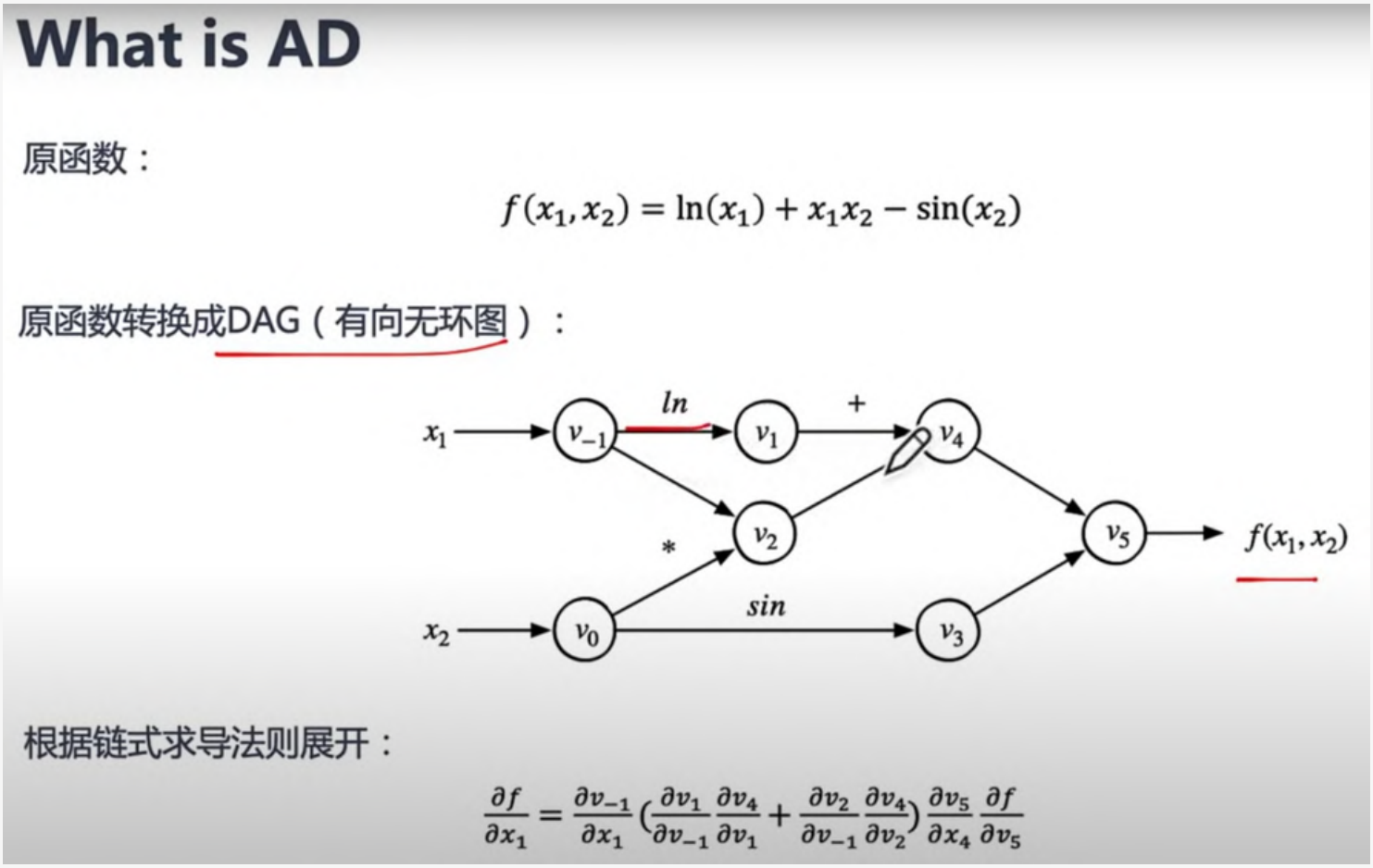
ADEV PAPER STUDY

ADEV是一個由MIT團隊所開發的**自動微分技術(AD)**的擴充。它將自動微分擴展到**處理隨機選擇**的模型(如PP)。

這是因為PP模型**並不是**參數固定的確定性函數，該模型會做出隨機選擇來模擬未知因素，且會在每次迭代中自動修改。如果嘗試在這些問題上使用一般AD，它們很容易給出**錯誤的答案**。

ADEV能夠準確處理做出隨機選擇的模型。這就把 AI 的優勢帶到了更廣泛的問題類別中，使人們能夠快速試驗能夠推理不確定情況的模型。

ADEV為**Forward Mode(前向模式)**的AD，適用於對單個輸入變量計算多個輸出變量的偏導數(因此模型的參數若過大容易降低計算效率)。它通過將每個輸入變量視為獨立的，沿著計算圖的正向運算來計算導數。



上圖截自:【自动微分】系列第三篇！微分的两种模式！前向微分和正向微分！对应反向传播！

THOUGHTS & REFLECTIONS

學習像 18.01x 這種大學線上課程對於一位剛升上高中的我確實非常有挑戰性，除了必須要適應**大學課堂的上課方式**，還需要熟悉許多**高中的數學知識**(如三角函數等)，**全英文的內容**對身為沒有出國留學過的公立高中學生更是一大困難。除此之外，在非國際學校裡要找到同樣理念及目標的同儕更是難上加難，這種種關卡在一開始都使沒有特殊背景的我感到十分挫折。但即使如此，對於微積分有興趣的我依然一步步地走過各種困難。

上圖為我2022暑假時的行事曆。

當時的我完全是在因緣際會下接觸到了PP與AD的領域，即使以我目前對於微分的知識還不足以讀懂ADEV的論文內容，我在 18.01x 所學的微積分知識也能夠讓我對於AD的概念較不陌生，也期待在日後能夠將ADEV應用於科展研究。

在一年的時間裡，我不僅完成了課程、做出了筆記、考完了AP、踏入了PP與AD的領域，獨立走出了與同儕不一樣的路，我希望能透過自身的經歷，影響我周圍的環境。